35. (a) We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields $-GMm/R + \frac{1}{2}mv^2 = 0$. We replace GM/R with $a_g R$, where a_g is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_g R + \frac{1}{2}v^2 = 0$. We solve for v:

$$v = \sqrt{2a_g R} = \sqrt{2(3.0\,\mathrm{m/s^2})(500 \times 10^3\,\mathrm{m})} = 1.7 \times 10^3\ \mathrm{m/s}\ .$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = \frac{1}{2}mv^2$. Suppose the particle is a distance h above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R+h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h}\ .$$

We replace GM with $a_g R^2$ and cancel m in the energy equation to obtain

$$-a_g R + \frac{1}{2}v^2 = -\frac{a_g R^2}{(R+h)}\ .$$

The solution for h is

$$\begin{aligned} h &= \frac{2a_g R^2}{2a_g R - v^2} - R \\ &= \frac{2(3.0\,\mathrm{m/s^2})(500 \times 10^3\,\mathrm{m})^2}{2(3.0\,\mathrm{m/s^2})(500 \times 10^3\,\mathrm{m}) - (1000\,\mathrm{m/s})^2} - (500 \times 10^3\,\mathrm{m}) \\ &= 2.5 \times 10^5\ \mathrm{m}\ . \end{aligned}$$

(c) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = -GMm/(R+h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h} = -\frac{GMm}{R} + \frac{1}{2}mv^2\ .$$

We substitute $a_g R^2$ for GM and cancel m, obtaining

$$-\frac{a_g R^2}{R+h} = -a_g R + \frac{1}{2}v^2\ .$$

The solution for v is

$$\begin{aligned} v &= \sqrt{2a_g R - \frac{2a_g R^2}{R+h}} \\ &= \sqrt{2(3.0\,\mathrm{m/s^2})(500 \times 10^3\,\mathrm{m}) - \frac{2(3.0\,\mathrm{m/s^2})(500 \times 10^3\,\mathrm{m})^2}{500 \times 10^3\,\mathrm{m} + 1000 \times 10^3\,\mathrm{m}}} \\ &= 1.4 \times 10^3\ \mathrm{m/s}\ . \end{aligned}$$